1.1 Introduction

In this chapter, our discussion begins by reviewing the standard terminology of radiation physics. This chapter discussed the most important physical and radiological quantities and the system of units by which they are measured. Due to its importance at many accelerators, the results of the special theory of relativity are reviewed. The energy loss by ionization and the multiple Coulomb scattering of charged particles is also summarized.

1.2 Review of Units, Terminology, Physical Constants, and Material Properties

1.2.1 Radiation Physics Terminology and Units

In order to develop an understanding of accelerator radiation physics, it is necessary to introduce the prominent quantities of importance and the units by which they are measured that are commonly used in accelerator radiation protection. Over the years various systems of units have been employed. Presently, there is a slow migration toward the use of the *Système Internationale* (SI) units. However, the practitioner needs to understand the interconnections of all of the units, both "customary" and SI, due to the great diversity of usage found in the scientific literature and in regulations.

energy: The unit of energy in common use when dealing with energetic particles is the electron volt (eV). 1 eV is equal to 1.602 x 10^{-12} ergs or 1.602 x 10^{-19} Joules. Multiples of these units in common use at accelerators are the keV (10^3 eV), MeV (10^6 eV), GeV(10^9 eV), and TeV (10^{12} eV). In the scientific literature, particle energies are almost always measured in these energy units rather than in the SI equivalent (i.e., Joules). Also, nearly always, the "energy" of a particle refers to the **kinetic energy** (see section 1.3).

absorbed dose: The energy absorbed per unit mass of material. It is usually denoted by the symbol *D*. The customary unit of absorbed dose is the **rad** while the *Système Internationale* (SI) unit of absorbed dose is the **Gray**. 1 rad is defined to be 100 ergs gram^{-1}, or, 6.24 x 10^{13} eV g^{-1}. One Gray (Gy) is defined to be 1 J kg^{-1} and is thus 100 rads. A Gray, then, is equal to 6.24 x 10^{15} eV g^{-1}. The concept of absorbed dose can be applied to any material. Thus it is commonly used to quantify both radiation exposures to human beings and the delivery of energy to materials and accelerator components.

dose equivalent: This quantity has the same physical dimensions as absorbed dose. It is used to take into account the fact that different particle types have biological effects which are enhanced, per given absorbed dose, over those due to the standard reference particles which are 200 keV photons. It is usually denoted by the symbol *H*. The customary unit is the **rem** while the SI unit is the **Sievert** (Sv). One Sievert is equal to 100 rem. The concept of dose equivalent is relevant only to radiation exposures received by human beings.

quality factor: This factor takes into account the relative enhancement in biological effects of various types of ionizing radiation. It is usually denoted by Q, and is used to connect H with D through the following equation;

$$H = QD. \tag{1.1}$$

Thus, H (rem) = QD (rads) or H (Sv) = QD (Gy). Q is dependent on both particle type and energy and, thus, for any radiation field its value is an average over all components. It is formally defined to have a value of unity for 200 keV photons. Q ranges from unity for photons, electrons of most energies, and high energy muons to a value as large as 20 for α–particles (i.e., ^{4}He nuclei) of a few MeV in kinetic energy. For neutrons, Q ranges from 2 to greater than 10. Although recent guidance by the International Council on Radiation Protection (ICRP) has recommended increased values of Q for neutrons (IC91), these increased values have yet to be adopted by regulatory authorities in the United States. Q is presently defined to be a function of **linear energy transfer** (LET), L. LET, crudely, is equivalent to **stopping power**, or rate of energy loss for charged particles and is conventionally expressed in units of keV μm^{-1}(see Section 1.4). All ionizing radiation ultimately manifests itself through charged particles so that LET is a good measure of localized radiation damage.

The value of Q commonly used is an average over the spectrum of LET present, weighted by the absorbed dose as a function of LET, $D(L)$;

$$\langle Q \rangle = \frac{\int_0^\infty dL Q(L) D(L)}{\int_0^\infty dL D(L)}. \tag{1.2}$$

Figures 1.1, 1.2, and 1.3 give the relationships between Q and LET and Q as a function of particle energy for a variety of particles and energies. The results shown in Fig. 1.2 are based upon ionization due to the *primary* particles only. For particles subject to the nuclear interaction, secondary particle production at higher energies will result in increased values of Q as a function of energy. For example for protons Q rises to a value of 1.6 at 400 MeV and a value of 2.2 at 2000 MeV (Pa73). The subject of relating operationally useful values of Q to the existing knowledge of radiobiological effects is a complex one, discussed at length elsewhere (NC90). In general, it is preferred to use the dose equivalent per fluence conversion factors discussed below.

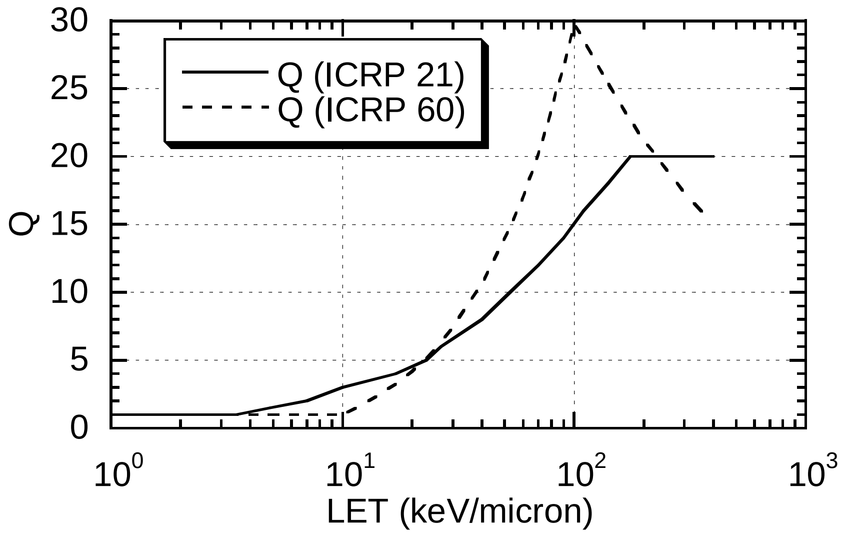

Fig. 1.1 Quality Factor, *Q*, of charged particles as a function of collision stopping power (LET) in water as recommended by the ICRP in Publication 21 (IC73) and later as revised in Report 60 (IC91).

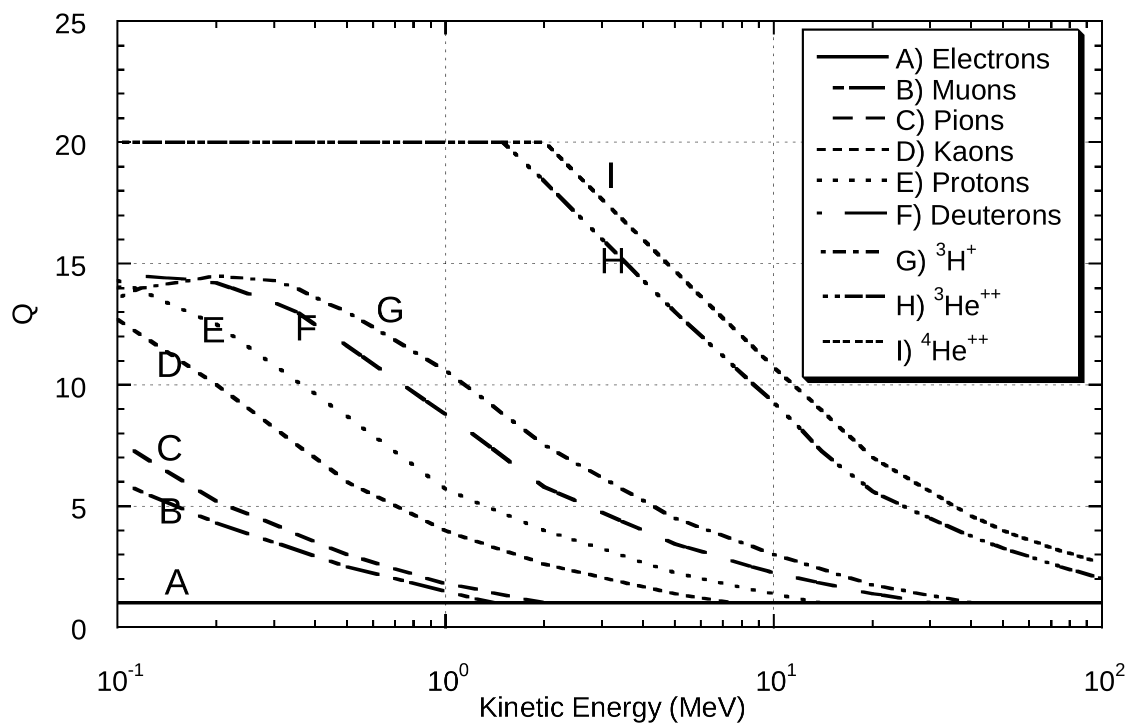

Fig. 1.2 Quality factors of charged particles as a function of energy, as recommended by the ICRP. [Adapted from (IC73)].

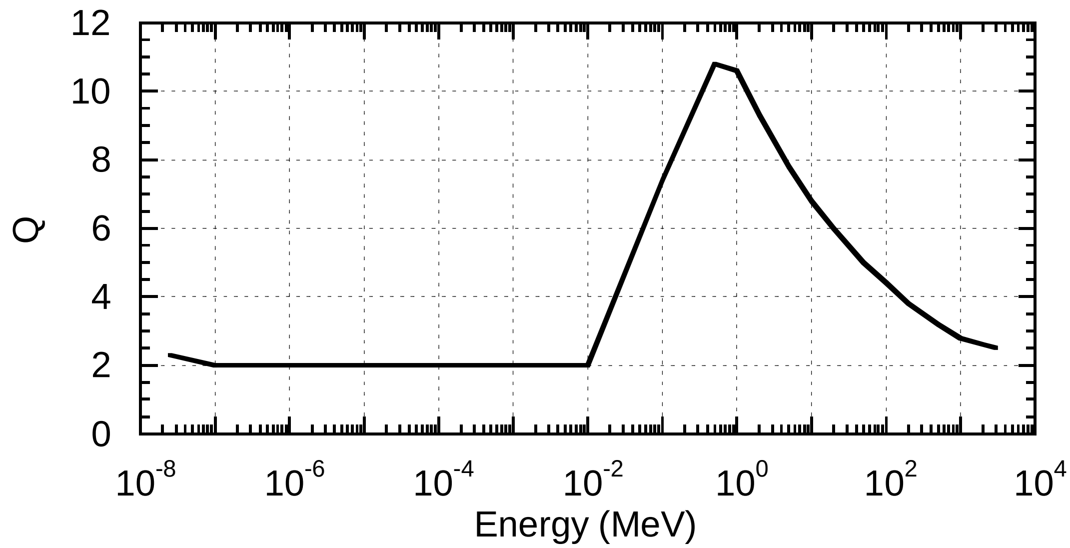

Fig. 1.3 Effective quality factor, Q, for neutrons as a function of neutron kinetic energy. The maximum dose equivalent divided by the absorbed dose where the maximum dose equivalent occurs (IC73) in human tissue. [Adapted from (Pa73).]

flux density: The number of particles that traverse a unit area in unit time. This quantity is generally denoted by the symbol, ϕ,

$$\phi = \frac{d^2 n}{dA dt}, \tag{1.3}$$

where $d^2 n$ is the differential number of particles traversing surface area element dA during time dt. For radiation fields where the constituent particles move in a multitude of directions, ϕ is the number of transversals of a sphere having a cross-sectional area dA. The units of flux density are $cm^{-2}s^{-1}$ (customary) and $m^{-2}s^{-1}$ (SI).

fluence: This quantity, denoted by Φ, is simply the integral over some time interval, $t_i < t < t_f$, of the flux density,

$$\Phi = \int_{t_i}^{t_f} dt \phi(t) \tag{1.4}$$

The units of fluence are, of course, inverse area. The reader is cautioned that other units of time such as hours, minutes, days, years, etc. are commonly found in the literature.

dose equivalent per fluence conversion factors: These factors include effects due to the finite thicknesses of the tissue and include effects due to secondary particles. Figures 1.4 and 1.5 are adapted from the tabulations of Schopper et al. (Sc90) and are appropriate for many particles commonly encountered. Muons, as will be seen later, are of particular importance at high energy accelerators. For these particles, the dose equivalent per fluence, *P*, has been found by Stevenson (St83) to be 40 fSv m^2 (i.e., 40 x 10^{-15} Sv m^2 or 25,000 muons cm^{-2} $mrem^{-1}$) for 100 MeV < E_μ < 200 GeV. At lower energies range-out of muons in the human body with consequential higher energy deposition gives a conversion factor of 260 fSv m^2 (3850 muons cm^{-2} per mrem). In principle, these values can be calculated for any particle. An example for more exotic particles is given in Fig. 1.6 for muon neutrinos, ν_μ's, results which might will be of potential importance at future accelerators presently under consideration (Co97 and Mo99).

For a radiation field containing a mixture of *n* different components (e.g., different particle types), one determines the dose equivalent, *H*, from

$$H = \sum_{i=1}^{n} \int_{E_{\min}}^{E_{\max}} dE P_i(E) \Phi_i(E) , \tag{1.5}$$

where $\Phi_i(E)$ is the fluence of particles of type *i* with energy between *E* and *dE* and $P_i(E)$ is the dose equivalent per unit fluence in appropriate units.

cross section: This quantity is an extremely important physical concept in describing particle interactions. The cross section represents the effective "size" of the atom or nucleus for some particular interaction. Consider a beam of particles of fluence Φ (particles cm^{-2}) incident on a thin slab of absorber of thickness *dx*. The absorbing medium has *N* atoms cm^{-3}. The number of incident particles that interact and are “lost” from the original fluence, $-d\Phi$, is given by:

$$-d\Phi = \sigma N \Phi dx \tag{1.6}$$

where σ is the cross section (cm^2). But, $N = \rho N_A/A$, where ρ is the density (g cm^{-3}), N_A is Avogadro's number (6.02 x 10^{23} mol^{-1}, see Table 1.1) and *A* is the atomic weight. Cross sections are often tabulated in units of **barns** where 1 barn is 10^{-24} cm^2. Submultiples such as the mb (10^{-3} barn, 10^{-27} cm^2) are commonly used. If only one physical process is present with no others are operative and if one starts with an initial fluence Φ_0, this integrates, after some distance *x* (cm), to

$$\Phi(x) = \Phi_o e^{-N\sigma x} . \tag{1.7}$$

linear absorption coefficient, μ, and its reciprocal, the **attenuation length**, λ: These quantities are given by:

$$\mu = N\sigma \text{ (cm}^{-1}) \qquad \lambda = 1/N\sigma \text{ (cm)}. \tag{1.8}$$

Sometimes the mass attenuation length, $\lambda_m = \rho/N\sigma$ (g cm^{-2}), is used where ρ is the density in g cm^{-3}. Unfortunately, in the literature λ is often used for λ_m so that one has to take care to understand the context to be sure to use the correct units. For high energy particles subject to the **strong,** or **nuclear, interaction**, λ, is commonly called the **interaction length.**

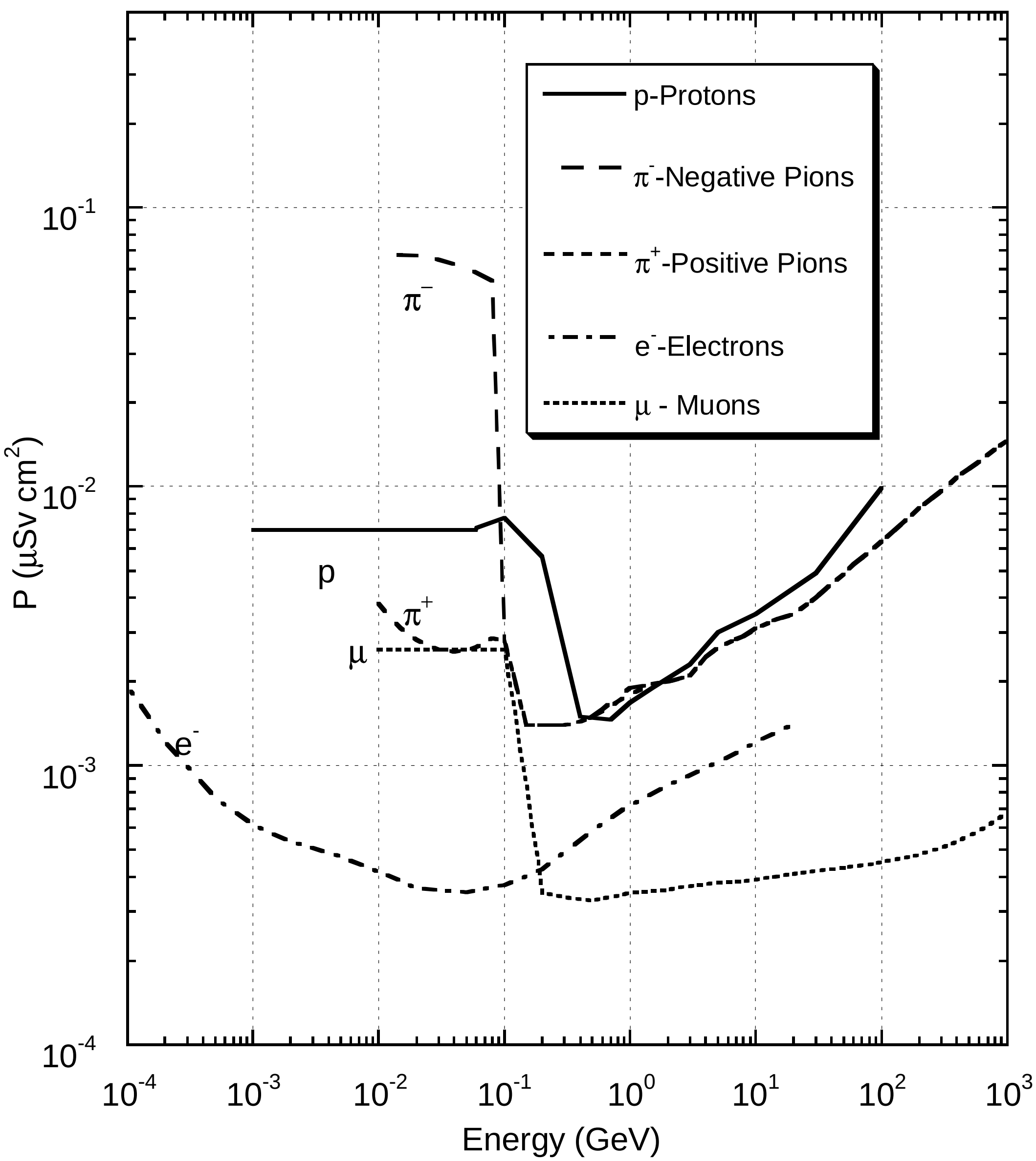

Fig. 1.4 Dose equivalent per fluence for various charged particles, *P*, as a function of energy. The curve for muons is valid for both negative and positively-charged muons. [Adapted from (Sc90).]

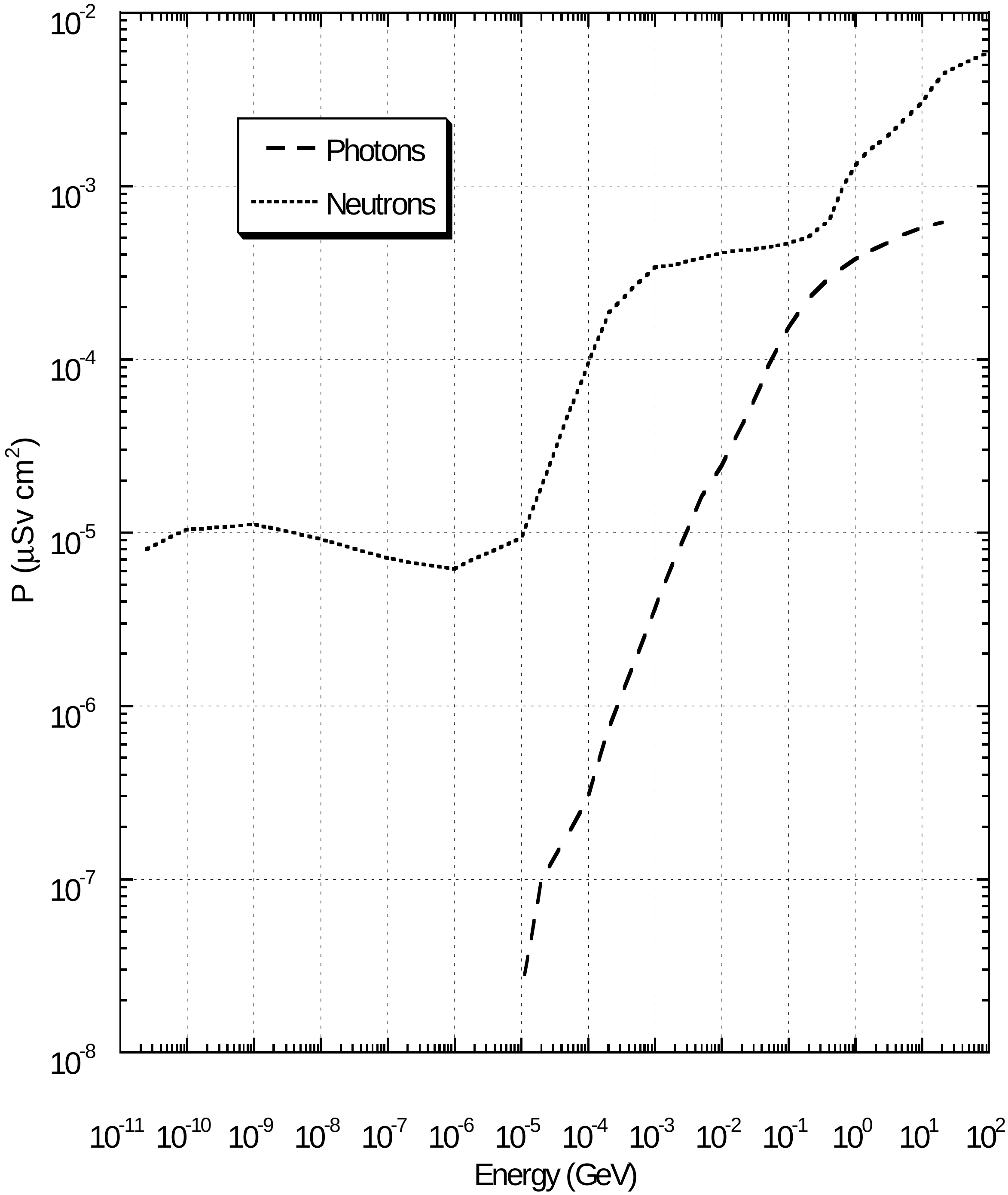

Fig. 1.5 Dose equivalent per fluence for photons and neutrons, *P*, as a function of energy. [Adapted from (Sc90).]

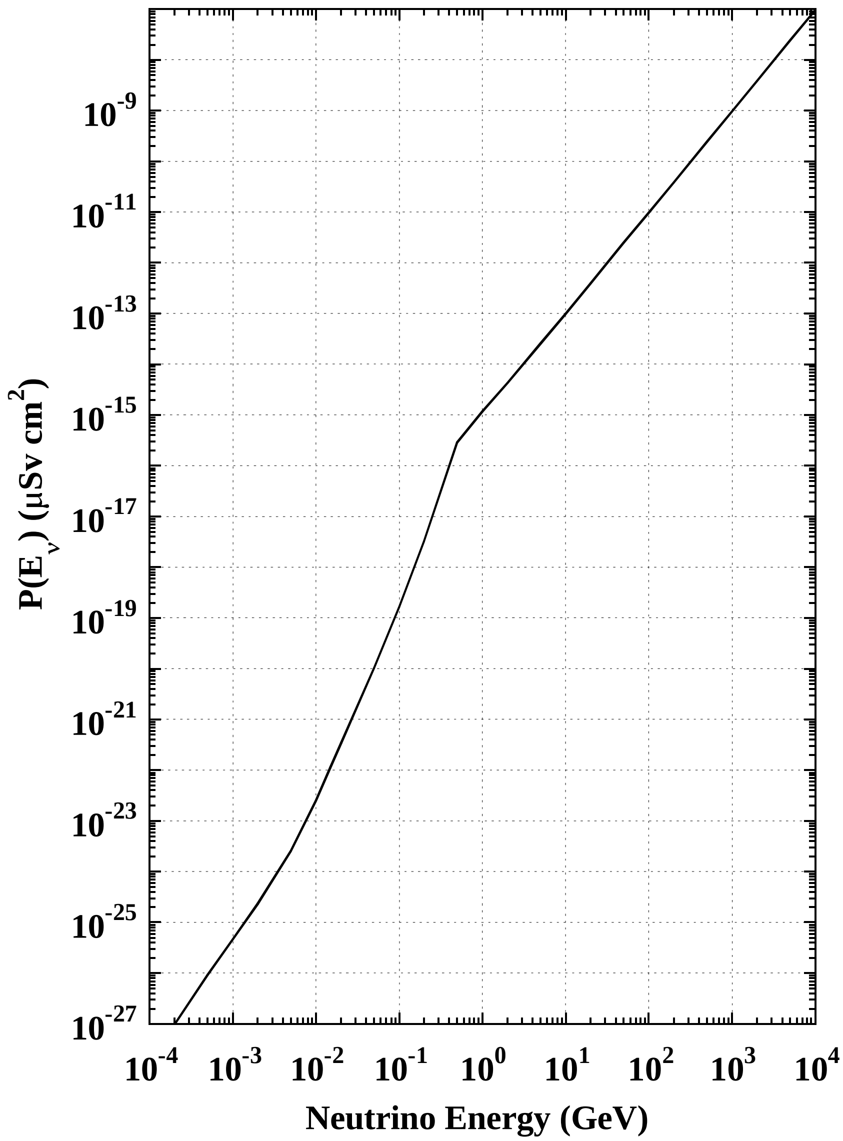

Fig. 1.6 Dose equivalent per fluence for muon neutrinos (ν_μ), *P*, as a function of energy. [Adapted from (Co97).]

1.2.2 Physical Constants and Atomic and Nuclear Properties

Tables 1.1 and 1.2 give physical constants and atomic and nuclear properties as tabulated by the Particle Data Group (PDG96)[1]. These tables are updated regularly and are republished every two years. A number of these constants and properties will be used throughout the rest of this text and in the solutions to the problems. Most of these quantities will be discussed subsequently and more details will be provided in subsequent chapters.

[1] The Particle Data Group maintains many tabulations on its website which are regularly updated with the latest values. The reference (PDG96) provides the link to this important source of information.

Table 1.1 Physical constants [Adapted from (PDG96)]

Quantity	Symbol, Equation	Value[a]
speed of light	c	2.99792458×10^{8} m s^{-1}
Planck constant	h	$6.6260755(40) \times 10^{-34}$ J s
Planck constant, reduced	$\hbar = h/2\pi$	$1.05457266(63) \times 10^{-34}$ J s $= 6.5821220(20) \times 10^{-22}$ MeV s
electron charge	e	$1.60217733(49) \times 10^{-19}$ C $= 4.8032068(15) \times 10^{-10}$ esu
useful constant	$\hbar c$	197.327053(59) MeV fm
useful constant	$(\hbar c)^2$	0.38937966(23) GeV^2 mbarn
electron mass	m_e	0.51099906(15) MeV/c^2 $= 9.1093897(54) \times 10^{-31}$ kg
proton mass	m_p	938.27231(28) MeV/c^2 $= 1.6726231(10) \times 10^{-27}$ kg = 1.007276470(12) u = 1836.152701(37) m_e
deuteron mass	m_d	1875.61339(57) MeV/c^2
unified atomic mass unit (u)	(mass ^{12}C atom)/12 = (1 g)/N_A	931.49432(28) MeV/c^2 $=1.6605402(10) \times 10^{-27}$ kg
permittivitiy of free space	ε_o	$8.854187817... \times 10^{-12}$ F m^{-1}
permeability of free space	μ_o, $[\varepsilon_o \mu_o = 1/c^2]$	$4\pi \times 10^{-7}$ N A^{-2}
fine structure constant[b]	$\alpha = e^2/4\pi\varepsilon_o \hbar c$	1/137.0359895(61)
classical electron radius	$r_e = e^2/4\pi\varepsilon_o m_e c^2$	$2.81794092(38) \times 10^{-15}$ m
electron compton wavelength	$\lambda\!\!\!^{-} = \hbar/m_e c = r_e/\alpha$	$3.86159323(35) \times 10^{-13}$ m
wavelength of 1 eV/c particle	hc/e	$1.23984244(37) \times 10^{-6}$ m
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	0.66524616(18) barn
Newtonian gravitational constant	G_N	$6.67259(85) \times 10^{-11}$ m^3 kg^{-1} s^{-2} $= 6.70711(86)\ \hbar c\ (\text{GeV}/c^2)^{-2}$
std. gravitational accel.	g	9.80665 m s^{-2}
Avogadro number	N_A	$6.0221367(36) \times 10^{23}$ mol^{-1}
Boltzmann constant	k	$1.380658(12) \times 10^{-23}$ J K^{-1} $= 8.617385(73) \times 10^{-5}$ eV K^{-1}
	1 barn	10^{-28} m^2 = 10^{-24} cm^2
	1 eV	$1.60217733(49) \times 10^{-19}$ J
	1 Gauss	10^{-4} Tesla
	1 erg	10^{-7} J
	1 fm	10^{-15} m
	1 atmosphere	760 torr = 1.01325×10^{5} N m^{-2}
	0^o C	273.15 ^{o}K

[a] The one-standard deviation uncertainties in the last digits are given in parentheses.
[b] This is the classic value, it becomes slightly larger at ultra large momentum transfers.

Table 1.2 Atomic and nuclear properties of materials [Adapted from (PDG96)]

Mat'l	Z	A	Nuclear total cross sect.[a] σ_T (barn)	Nuclear inelastic cross sect.[a] σ_{in} (barn)	Nuclear collision length[b] λ_T (g/cm^2)	Nuclear interaction length[b] λ_{in} (g/cm^2)	Min. stopping power[c] dE/dx (MeV/g/cm^2)	Radiation X_o (g/cm^2)	Length X_o (cm) () is for gas	Density[d] ρ (g/cm^3) ( ) or [ ] for gas (g/l)
H_2	1	1.01	0.0387	0.033	43.3	50.8	4.12	61.28	865	0.0708(0.090)
D_2	1	2.01	0.073	0.061	45.7	54.7	2.07	122.6	757	0.162[0.179]
He	2	4.00	0.133	0.102	49.9	65.1	1.94	94.32	755	0.125[0.179]
Li	3	6.94	0.211	0.157	54.6	73.4	1.58	82.76	155	0.534
Be	4	9.01	0.268	0.199	55.8	75.2	1.61	65.19	35.3	1.848
C	6	12.01	0.331	0.231	60.2	86.3	1.78	42.70	18.8	2.265[e]
N_2	7	14.01	0.379	0.265	61.4	87.8	1.82	37.99	47.0	0.808[1.25]
O_2	8	16.00	0.420	0.292	63.2	91.0	1.82	34.24	30.0	1.14[1.43]
Al	13	26.98	0.634	0.421	70.6	106.4	1.62	24.01	8.9	2.70
Si	14	28.09	0.660	0.440	70.6	106.0	1.66	21.82	9.36	2.33
Ar	18	39.95	0.868	0.566	76.4	117.2	1.51	19.55	14.0	1.40[1.78]
Fe	26	55.85	1.120	0.703	82.8	131.9	1.48	13.84	1.76	7.87
Cu	29	63.55	1.232`	0.782	85.6	134.9	1.44	12.86	1.43	8.96
Ge	32	72.59	1.365	0.858	88.3	140.5	1.40	12.25	2.30	5.323
W	74	183.85	2.767	1.65	110.3	185	1.16	6.76	0.35	19.3
Pb	82	207.19	2.960	1.77	116.2	194	1.13	6.37	0.56	11.35
U	92	238.03	3.378	1.98	117.0	199	1.09	6.00	0.32	18.95
Air					62.0	90.0	1.82	36.66	(30420)	(1.205)[1.293]
H_2O					60.1	84.9	2.03	36.08	36.1	1.00
Shielding concrete[f]					67.4	99.9	1.70	26.7	10.7	2.5
SiO_2 (quartz)					67.0	99.2	1.72	27.05	12.3	2.64
NaI					94.8	152	1.32	9.49	2.59	3.67
Polystyrene, scintillator (CH)					58.4	83.0	1.95	43.8	42.4	1.032
Polyethylene (CH_2)					56.9	78.8	2.09	44.8	47.9	0.92-0.95
Mylar ($C_5H_4O_2$)					60.2	85.7	1.86	39.95	28.7	1.39
CO_2					62.4	90.5	1.82	36.2	(18321)	[1.977]
Methane (CH_4)					54.7	74.0	2.41	46.5	(64850)	0.424[0.717]
Ethane (C_2H_6)					55.7	75.7	2.30	45.7	(34035)	0.509(1.356)
NaF					66.78	97.57	1.69	29.87	11.68	2.558
LiF					62.0	88.24	1.66	39.25	14.91	2.632

[a]These are energy dependent. The values quoted are for the high energy limit. The inelastic cross section is obtained by subtracting the elastic and quasi-elastic cross sections from the total cross section.

[b]These quantities are the mean free path between all collisions (λ_T) or inelastic interactions (λ_{in}) and are also energy-dependent The values quoted are for the high energy limit.

[c]This is the minimum value of the ionization stopping power for heavy particles. It is calculated for pions and the results are slightly different for other particles.

[d]For substances that are gases at room temperature, values at 20 ^oC and 1 atmosphere pressure are given in parentheses (grams/liter) while values at STP are given in square brackets [grams/liter]. Values without ( ) or [ ] are for cryogenic liquids at the boiling point at 1 atmosphere pressure (g cm^{-3}).

[e]The tabulated values are for pure graphite; industrial graphite may vary between 2.1-2.3 g cm^{-3}.

[f]This is for standard shielding blocks, typical composition of O_2 (52%), Si (32.5%), Ca (6%), Na (1.5%), Fe (2%), Al (4%), plus reinforcing iron bars.

1.3 Summary of Relativistic Relationships

The results of the special theory of relativity are quite evident at most accelerators. In this section, the important conclusions are reviewed.

The **rest energy**, W_o, of a particle of rest mass m_o is given by

$$W_o = m_o c^2 , \tag{1.9}$$

where c is the velocity of light. The **total energy** in free space, W, is given by

$$W = mc^2 = \frac{m_o c^2}{\sqrt{1-\beta^2}} = \gamma m_o c^2 , \tag{1.10}$$

where $\beta = v/c$ and v is the velocity of the particle in a given frame of reference. The relationship between the quantities β and γ is obvious. Similarly, the **relativistic mass**, m, of a particle moving at velocity β is given by

$$m = \frac{m_o}{\sqrt{1-\beta^2}} = \gamma m_o . \tag{1.11}$$

The **kinetic energy**, E, is then,

$$E = W - W_o = (m - m_o)c^2 \qquad \text{and} \tag{1.12}$$

$$\beta = \sqrt{1-\left(\frac{W_0}{W}\right)^2} . \tag{1.13}$$

The **momentum**, p, of a particle in terms of its relativistic mass, m, and velocity, v, is,

$$p = mv = m\beta c = \frac{1}{c}\sqrt{E(E+2W_0)} = \frac{E}{c}\sqrt{1+\frac{2W_0}{E}} , \tag{1.14}$$

so that at high energies, $p \approx E/c \approx W/c$.

It is usually most convenient to work in a system of units where energy is in units of eV, MeV, etc. Velocities are then expressed in units of the speed of light (β), momenta are expressed as energy divided by c (e.g., MeV/c, etc.), and masses are expressed as energy divided by c^2 (e.g., MeV/c^2, etc.). In these units, the total energy, W, and the relativistic mass, m, are equivalent. One thus avoids the explicit evaluation of c, or c^2.

The **decay length** at a given velocity of a particle with a finite meanlife (at rest), τ, is given by $\beta\gamma c\tau$, where relativistic time dilation is taken into account by inclusion of the factor γ. The product of the speed of light and the meanlife, $c\tau$ is often tabulated. The decay length is the mean distance traveled by a particle in vacuum prior to its decay. This length must be distinguished from that called the **decay path**. The decay path represents a distance in space in which a given particle is allowed to decay with no or minimal competition from other effects such as by scattering or absorption. Thus, the decay length is determined by the basic physics of the decay process while the decay path is defined by the physical configuration of the accelerator components present.

1.4 Energy Loss by Ionization and Multiple Coulomb Scattering

1.4.1 Energy Loss by Ionization

For moderately relativistic particles, the mean rate of energy loss (**stopping power**) is given approximately by (PDG96);

$$-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left\{\frac{2m_e c^2\gamma^2\beta^2}{I}\right\} - \beta^2 - \frac{\delta}{2}\right] \text{ (MeV cm}^2\text{g}^{-1}), \quad (1.15)$$

where N_A is Avogadro's number, Z and A are the atomic number and weight of the material traversed, z is the charge state of the projectile in units of electron charge, m_e and r_e are the rest mass and "classical radius" of the electron (see Table 1.1) and I is the ionization constant. For $Z > 1$, $I \approx 16Z^{0.9}$ eV while for diatomic hydrogen (H_2), $I = 19$ eV. β and γ are as defined in Section 1.3. δ is a small correction factor that approaches 2 lnγ. Substituting constants,

$$-\frac{dE}{dx} = 0.3071 z^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left\{\frac{2m_e c^2\gamma^2\beta^2}{I}\right\} - \beta^2 - \frac{\delta}{2}\right] \text{ (MeV cm}^2\text{g}^{-1}). \quad (1.16)$$

This is the stopping power[2] due to ionization, the process in which a charged particle transfers its energy to atomic electrons in the absorbing medium. In these units, the dependence upon the absorbing material is slowly-varying given the fact that I appears only in the logarithmic term and the ratio Z/A ranges between 0.4 to 0.5 over most of the periodic table for stable nuclides. Thus, for a given projectile charge z the value of the stopping power, dE/dx is most strongly dependent on β. A broad minimum is found at a value of $\gamma = 3.2$. At this value of γ, the particles are said to be **minimum ionizing** and the corresponding minimum stopping powers are listed in Table 1.2.

The absorption of the energy of charged particles by ionization is characterized by a parameter called the **range**, R, in material. The range is the length of the path through followed by the particle while it is losing its energy. Simplistically one might think that one could calculate the value of R by a simple integration of reciprocal of the stopping

[2] The argument of the logarithmic term of Eqs (1.15) and (1.16) must be dimensionless. Hence, the rest energy of the electron, $m_e c^2$, and I must be in the same units (e.g., both in eV).

power. However, as the particles lose energy by ionization and thus slow down, other effects at very low energies become important that are not included in Eq. (1.16). It is prudent, therefore, to consult explicit tabulations to determine the particle ranges. For charged particles much more massive than electrons, the trajectory through the material to first approximation is a straight line modified only by multiple Coulomb scattering (see Section 1.4.2) since the mass of the moving particle is so much larger than the mass of the atomic electrons. For a moving electron, the range is the sum of many line segments through the material since its mass is identical to that of the atomic electrons encountered with the consequence that the individual angular deflections are much larger. As shall be seen in Section 3.2.2, for electrons the loss of energy in matter due to the radiation of photons as the kinetic energy of the electron increases rapidly becomes much more important than the ionization stopping power or the range. The situation is different for particles such as protons that participate in the strong (i.e., nuclear) interaction. For these particles, as the kinetic energy of the particle increases, the absorption of the particles through strong interaction processes has a high probability of absorbing the particles prior to their depositing all of their energy by ionization. This will be discussed further in Section 4.2.1. Figures 1.7 and 1.8 give stopping power and range values as a function of momentum or energy for common high energy particles and for some light ions, respectively. Detailed tables of the values of stopping power and ranges for many heavy ions have been given by Northcliffe and Schilling (No70). Also, the Monte Carlo computer code SRIM is currently easily obtained and may be used to generate similar tables as well as do simulations of protons or heavy charged ions interacting with elemental or compound materials (Zi96).

For muons (μ's) the situation is rather unique. The muon rest energy is 105.7 MeV, its meanlife $\tau = 2.20 \times 10^{-6}$ s, and the meanlife times the speed of light is $c\tau = 658.6$ m. Due to their large rest mass compared to that of the electron and the fact that these particles, to first order, do not participate in the strong interaction, muons tend to penetrate far distances in matter without being absorbed by other mechanisms. Muons, due to their heavier masses, are also far less susceptible to radiative effects. Thus, over a very large energy domain, the principal energy loss mechanism is that of ionization. This, as shall be seen later, makes the shielding of muons matter of considerable importance at high energy accelerators. The range-energy relation of muons is given in Fig. 1.9. At high energies ($E_\mu > 100$ GeV), the distribution of the ranges of individual muons about the mean range, called the **range straggling**, becomes severe (Va87). Also, above a muon energy of several hundred GeV (in, say, iron), radiative losses begin to dominate such that the stopping power, *dE/dx*, is given by

$$\frac{-dE}{dx} = a(E) + b(E)E, \tag{1.17}$$

where *a(E)* is the collisional ionization energy loss given by Eq. (1.16) (≈ 0.002 GeV cm^2 g^{-1}), and *b(E)* is the radiative coefficient for *E* in GeV. The latter parameter separated into contributions from the important physical mechanisms is plotted in Fig. 1.10.

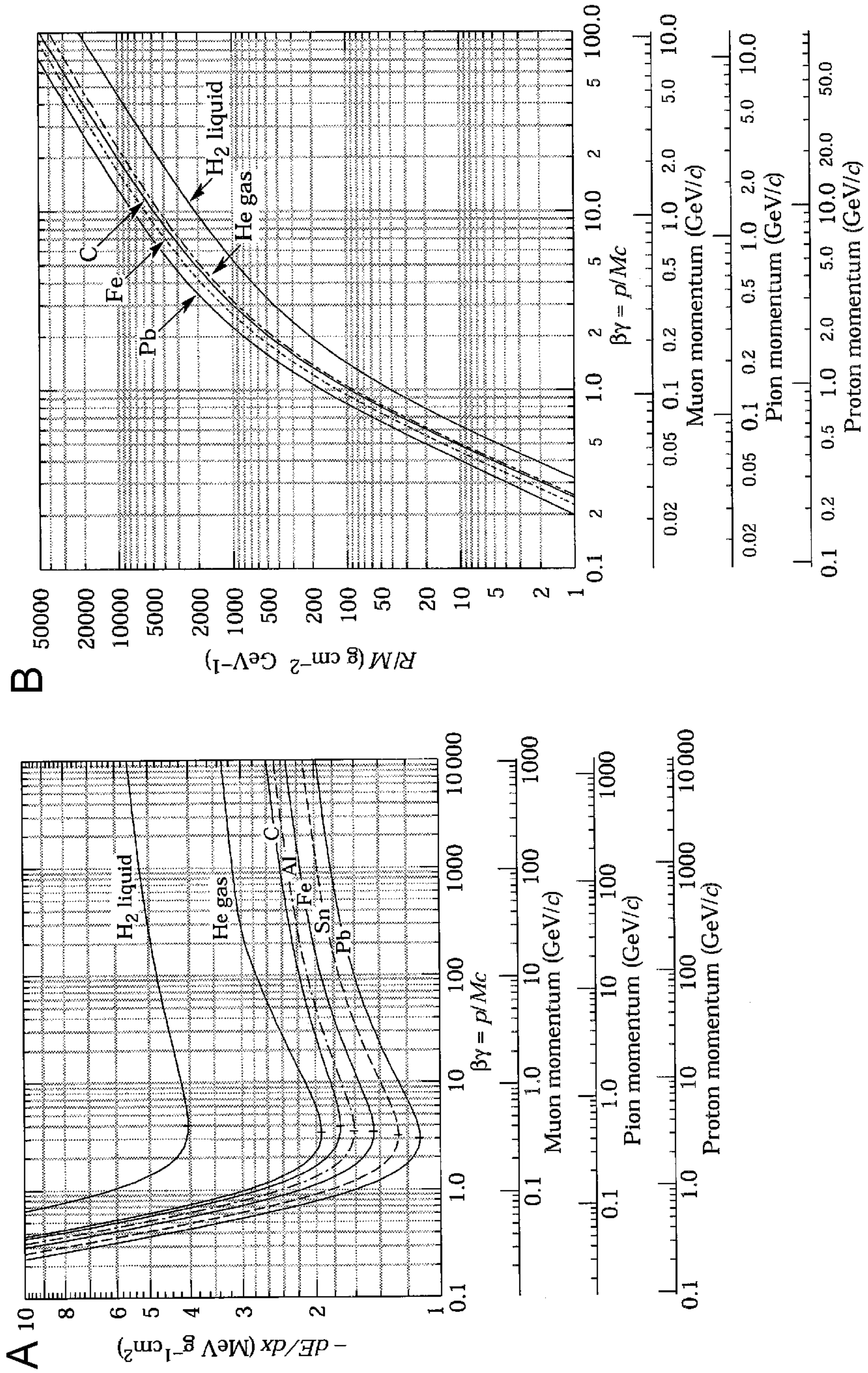

Fig. 1.7 **A.** Stopping power in various media as a function of particle momenta. **B.** Range of heavy charged particles in various media. The abscissa of these plots are scaled to the ratio of particle momenta, p, to particle rest mass, M. [Reproduced from (PDG96).]

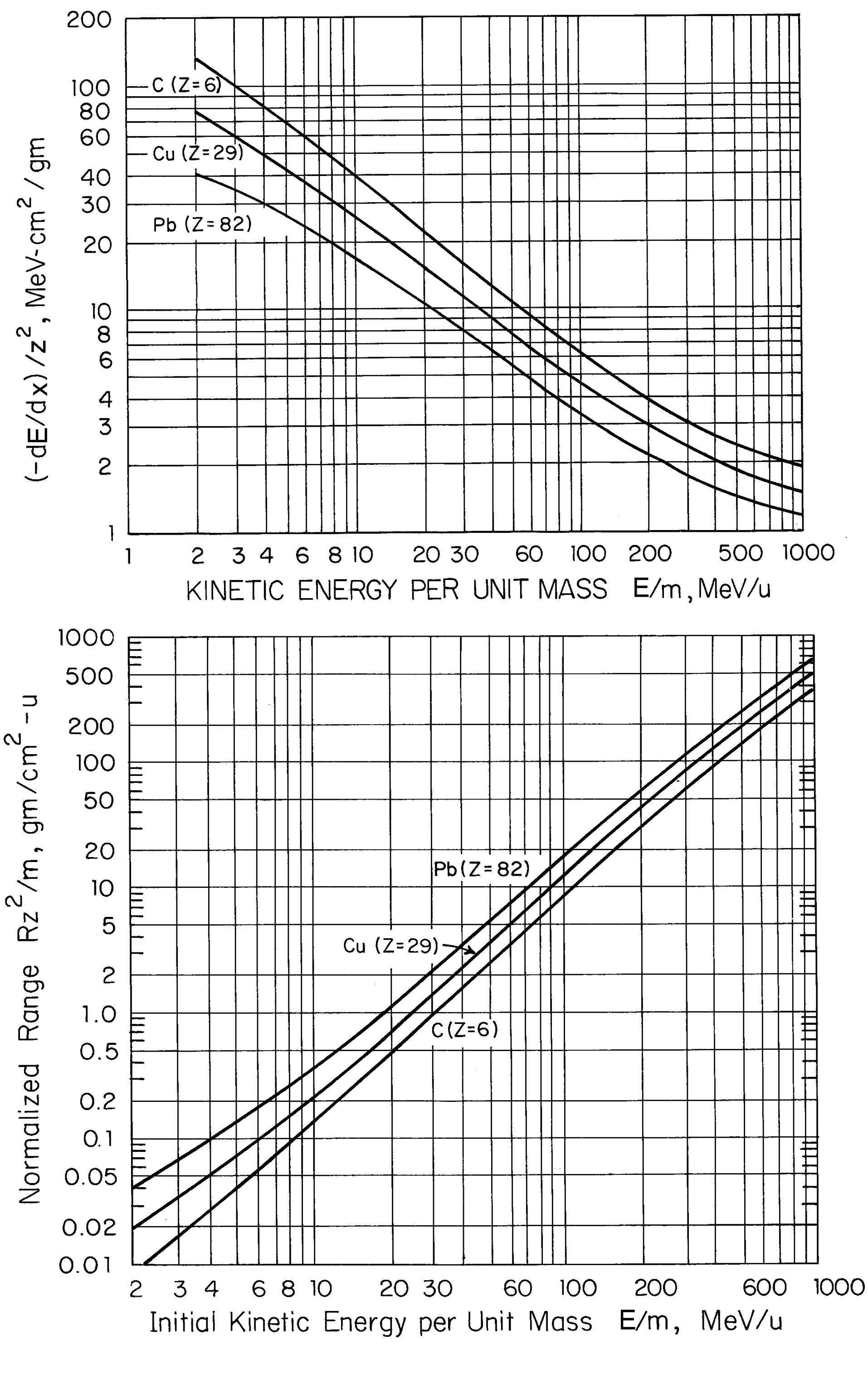

Fig. 1.8 Stopping power (**top**) and ranges (**bottom**) for protons in three different materials. These curves can be used for other incident particles by taking their atomic number, *z*, and mass, *m*, into account. The incident energy is thus expressed as the specific kinetic energy, *E/m*. The curves are approximately correct except at the very lowest energies where charge exchange effects can be important. The results are most valid for projectile mass, $m \leq 4$ [Adapted from (En66).]

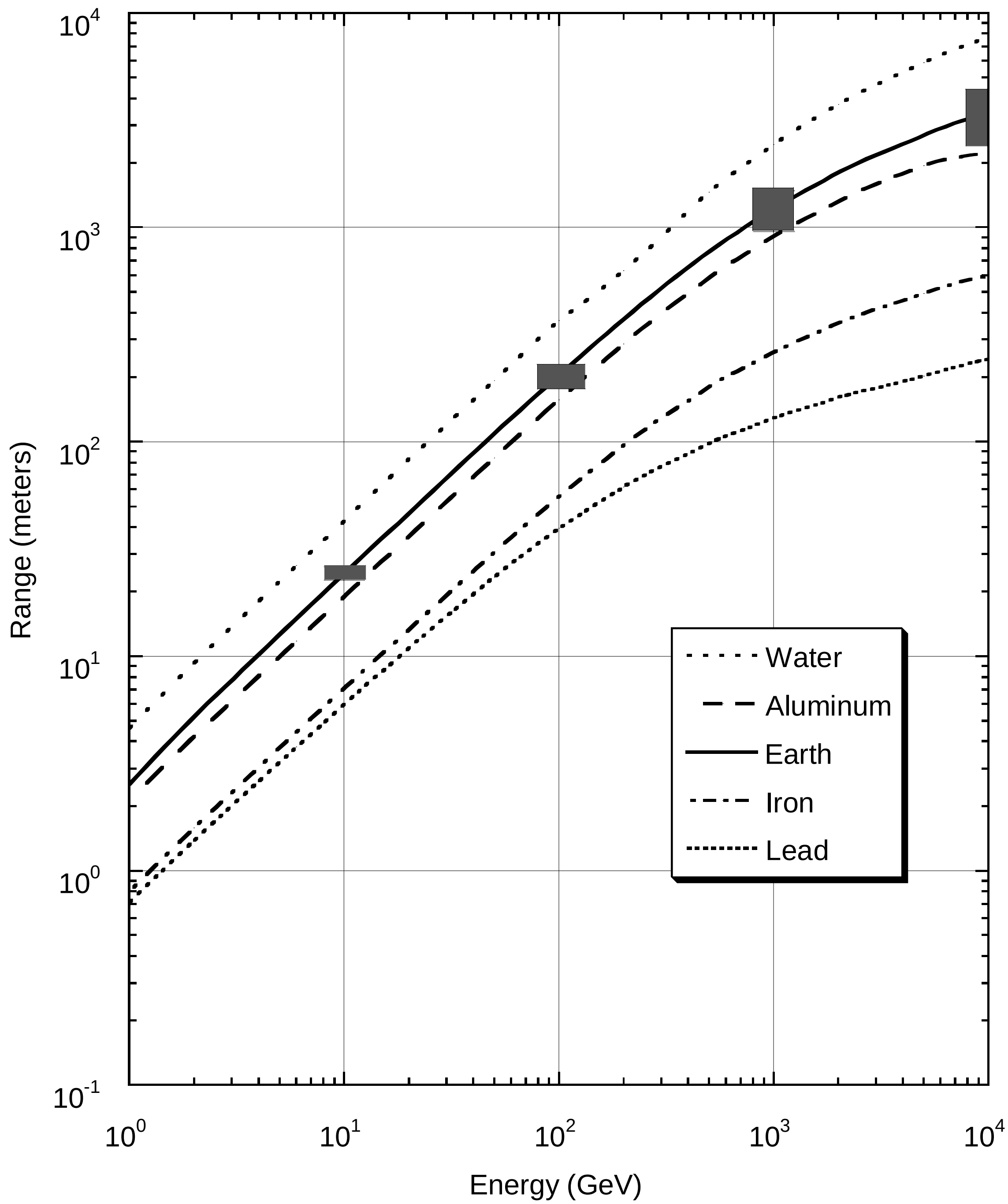

Fig. 1.9 Range-energy curves for muons in various materials. On the curve labeled "Earth", the gray boxes are indicative the approximate spread in the range due to range straggling at one standard deviation at the indicated muon energy. The density of "earth" was taken to be 2.0 g cm^{-2}. [Adapted from (Sc90).]

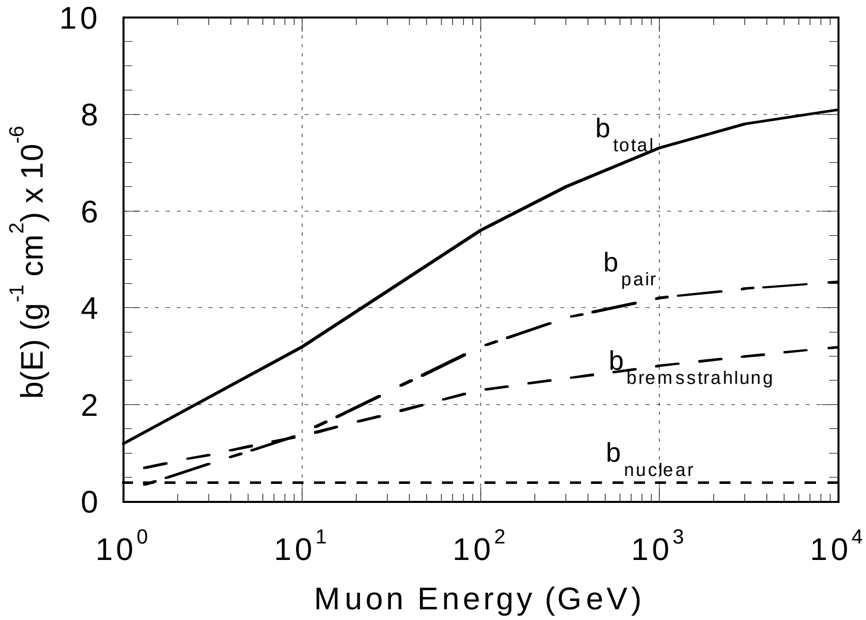

Fig. 1.10 Contributions to the fractional energy loss by muons in iron due to e^+e^- pair production, bremsstrahlung, and photonuclear interactions. See Eq (1.17). [Adapted from (PDG96).]

The mean range, R_μ, of a muon of kinetic energy E_o, is approximated by

$$R_\mu \approx \frac{1}{b}\ln\left(a + bE_0\right). \tag{1.18}$$

Muon range straggling (Va87) is chiefly due to the fact that, for muon kinetic energies greater than about 100 GeV, electron-positron pair production, bremsstrahlung, and deep inelastic nuclear reactions become the dominant energy loss mechanisms. The cross sections for the latter two mechanisms are such that only a few interactions can be expected. Although these processes have low probabilities, when they do occur they involve large energy losses and thus have quite significant effects. Tables 1.3 and 1.4 give fractional energy loss and comparisons of muon ranges at high energies for different physical mechanisms. Here, the straggling is very important since shielding calculations based upon using the mean range values can lead to significant *underestimates* of the fluence of muons which can penetrate the shield.

Table 1.3 Fractional energy loss of muons in soil (ρ = 2.0 g cm^{-3}). The fractions of the total energy loss due to the four dominant energy loss mechanisms are given. [Adapted from (Va87) and (Sc90).]

Energy (GeV)	Ionization	Bremsstrahlung	Pair production	Deep inelastic nuclear scattering
10	0.972	0.037	8.8 x 10^{-4}	9.7 x 10^{-4}
100	0.888	0.086	0.020	0.0093
1000	0.580	0.193	0.168	0.055
10,000	0.167	0.335	0.388	0.110

Table 1.4 Comparison of muon ranges (meters) in heavy soil (ρ = 2.24 g cm^{3}) [Adapted from (Va87) and (Sc90).]

Energy			Mean Ranges from *dE/dx* in Heavy Soil (meters)		
(GeV)	Mean Range (meters)	Standard Deviation (meters)	All Processes	Coulomb Losses Only	Coulomb & Pair Production Losses
10	22.8	1.6	21.4	21.5	21.5
30	63.0	5.6	60.3	61.1	60.8
100	188	23	183	193	188
300	481	78	474	558	574
1000	1140	250	1140	1790	1390
3000	1970	550	2060	5170	2930
10,000	3080	890	3240	16,700	5340
20,000	3730	1070			

1.4.2 Multiple Coulomb Scattering

Multiple Coulomb scattering from nuclei is an important effect in the transport of charged particles through matter. A charged particle traversing a medium is deflected by many small-angle scattering events and only occasionally by ones involving large-angle scattering. The small-angle scattering events are largely due to Coulomb scattering from nuclei so that the effect is called multiple Coulomb scattering. This simplification is not quite correct for hadrons since it ignores the contribution of strong interactions to multiple scattering. For purposes of discussion here, a Gaussian approximation adequately describes the distribution of deflection angles of the final trajectory compared with the incident trajectory for all charged particles. The distribution as a function of deflection angle, θ, is as follows:

$$f(\theta)d\theta = \left(\frac{d\theta}{\theta_0\sqrt{2\pi}}\right)\exp\left(-\frac{\theta^2}{2\theta_0^2}\right). \tag{1.19}$$

The mean width in space of the projected angular distribution, θ_o, is given by

$$\theta_o = \frac{13.6z}{pc\beta}\sqrt{\frac{L}{X_o}}\left\{1+0.038\ln\left(\frac{L}{X_o}\right)\right\} \text{ (radians)} \qquad (1.20)$$

where z is the charge of the projectile, in units of the charge of the electron, p is the particle momentum in MeV/c and L is the shield thickness in the same units as the quantity X_o (PDG96). X_o is a material-dependent parameter, to be discussed further in Section 3.2.2 called the **radiation length**. The radiation length is approximately given by

$$X_o = \frac{716.4A}{Z(Z+1)\ln\left(\frac{287}{\sqrt{Z}}\right)} \text{ (g cm}^{-2}\text{)}, \qquad (1.21)$$

where Z and A are the atomic number and weight of the material medium, respectively.

1.5 Radiological Standards

While the discussion of radiological standards is not a topic of great emphasis in this text, some mention of it seems to be appropriate. Standards or limits on occupational and environmental exposure to ionizing radiation are now instituted worldwide. In general, individual nations, or sub-national entities, incorporate guidance provided by international or national bodies into their laws and regulations. The main international body that develops radiological standards is the International Commission on Radiation Protection (ICRP). In the United States, the major national body, chartered by the U. S. Congress is the National Council on Radiation Protection and Measurements (NCRP). In the U.S. the Environmental Protection Agency (EPA) is the primary federal agency for establishing basic radiological standards. These standards are further implemented by the U.S. Department of Energy (DOE) for its facilities, and by the Nuclear Regulatory Commission (NRC) for its licensees. The regulation of accelerator facilities varies considerably between individual states and some local jurisdictions.

Problems

1. a) Express 1 kilowatt (1 kW) of beam power in GeV s^{-1}.

   b) To how many singly charged particles per second does 1 ampere of beam current correspond?

   c) Express an absorbed dose of 1 Gy in GeV kg^{-1} of energy deposition.

2. Which has the higher quality factor, a 10 MeV (kinetic energy) α-particle or a 1 MeV neutron? Write down the quality factors for each particle.

3. Calculate the number of ^{12}C and ^{238}U atoms per cm^3 of solid material.

4. Calculate the velocity and momenta of a 200 MeV electron, proton, iron ion, π^+, and μ^+. The 200 MeV is kinetic energy and the answers should be expressed in units of the speed of light (velocity) and MeV/c (momenta). Iron ions have an isotope-averaged mass of 52021 MeV (A = 55.847 x 931.5 MeV/amu). The π^+ mass is 140 MeV and the μ^+ mass = 106 MeV. Do the same calculation for 20 GeV protons, iron ions, and muons. It is suggested that these results be presented in tabular form. Make general comments on the velocity and momenta of the particles at the two energies. (The table may help you notice any algebraic errors that you have made.)

5. Calculate the mass stopping power of a 20 MeV electron (ionization only) and a 200 MeV proton in ^{28}Si.

6. Calculate the fluence of minimum ionizing muons necessary to produce a dose equivalent of 1 mrem assuming a quality factor = 1 and that <u>tissue</u> is equivalent to <u>water</u> for minimum ionizing muons. (Hint: use Table 1.2.) Compare with the results given in Fig. 1.4 for high energies.